

Modeling of viscous damping of perforated planar microstructures. Applications in acoustics

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The paper contains an analysis of the viscous damping in perforated planar microstructures that often serve as backplates or protecting surfaces in capacitive microsensors. The focus of this work is on planar surfaces containing an offset system of periodic oval holes or its limit cases: a system of circular holes or of slits. The viscous damping is calculated as the sum of squeeze film and the holes' resistances. The optimum number of holes is determined which minimizes the total viscous damping for a given percentage of open area. Graphs and formulas are provided for designing these devices. In the case the open area is higher than 15% the numerical results show that the influence of the holes' geometry (circular or oval) has a slight influence on viscous damping. As the planar structures containing oval holes assure a better protection against dust particles and water drops, they should be preferred in designing protective surfaces for microphones working in a natural environment. The obtained results also can be applied in designing other MEMS devices that use capacitive sensing such as accelerometers, micromechanical switches, resonators, and tunable microoptical interferometers. © 2004 Acoustical Society of America. [DOI: 10.1121/1.1798331]

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I. INTRODUCTION

The work of electrostatic transducers is based on the capacitive detection principle. Hence, two important parts of such a device are the diaphragm, in the case of microphones, or the proof mass in the case of accelerometers, and the backplate electrode. The small space between these elements is filled with fluid (air). This system of plates with the associated air layer between them will be referred to as a planar microstructure. On the other hand, in order to protect the diaphragm from external damages a certain perforated planar microstructure (for example, some gratings) may be placed in the front of the microphone at small distance from the diaphragm. In some design solutions a single perforated planar microstructure is used for both functions.

The gas flow between the closed parallel plates causes viscous and inertial forces. These forces decrease the microphone performance due to the noise associated with them, and their simulation is important in predicting the device behavior. The aim of such a simulation is to design a planar microstructure which maximizes the capacitance and penetration of sound waves and minimizes the parasitic fluid action (the viscous damping). In the case of membrane damping of the electrostatic transducer (sensor, microphone), the number of holes must determine the optimal required damping due to viscous losses in the air gap needed to obtain a flat frequency response.

The fluid dynamics can be described in terms of classical Navier–Stokes equations for an incompressible fluid. The special geometry of the problem, namely the small space between the plates (squeeze-film flow), yields some simplifications of the equations similar to the lubrication approxi-

mation in classical hydrodynamics. Thus, the effect of the inertial terms, other than the time derivative of velocities, is negligible. The basic equation resulting from this analysis is a Poisson equation; the source term contains a factor characterizing the frequency dependence, another one depending upon geometry, and also the normal velocity of the diaphragm.

This analysis provides first the squeeze-film damping of the microstructure. In order to decrease the viscous damping we consider a repetitive pattern of holes on the backplate, each of them having its own domain of influence (also called a “cell”). In most applications the holes are considered circular.¹ The simulation in this case is simpler and in certain applications it gives the desirable data to be used for design purpose. An approximate formula for squeeze-film damping, for this situation, was obtained by means of some hydraulic considerations by Škvor.² Our studies revealed that other geometrical shapes of the holes can be used, such as ellipses and ovals. It is shown that the use of noncircular shapes of the holes can provide a reduction in damping for the same percent open area. As the elongated ovals (and the slits as a limit case) are more likely to be obtained technologically and can be used also for protecting purposes, in this paper we shall focus on the study of the viscous damping in this type of geometry of the planar microstructures. (In fact, a planar structure involving slits was used previously in³ designing accelerometers.)

When the squeezing film and plate thicknesses are comparable, as is the case of the surface-micromachined planar microstructures, introduction of each new hole is associated with a “hole resistance” which is added to the squeeze-film damping.⁴ The calculation of the hole resistance is obtained by modeling the flow in a hole as a Poiseuille flow in a pipe, described again by a Poisson equation for velocity. As the two components of the total viscous damping have opposite

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variations with respect to the increase of number of holes, there is an optimum number of holes which assures an equilibrium between squeeze-film damping and holes resistance which provides a minimum to the viscous damping coefficient.

The developed method is applied, first, to the case of circular holes. The case of microplanar structures containing slits is considered in Sec. IV. In both cases there are analytical asymptotic formulas for squeeze-film damping, the holes resistance, and the optimum number of holes, assuring a minimum viscous damping for the microstructure. In Sec. V the case of planar microstructures having a repetitive pattern of oval holes is considered. The formulas for squeeze-film damping and the holes resistance involve two coefficients which have to be determined numerically by solving two boundary-value problems for Poisson's equations. By an optimization technique, formulas are obtained giving the optimum number of oval holes and the associated minimum value of the total damping coefficient. These formulas contain two numerical coefficients, N^* and B^* , depending only upon the holes' geometry. The numerical results show that while for small values of the ratio of the open area to the total area, AR, all geometrical parameters of the assigned oval pattern are important in determining the coefficient, B^* , entering in minimum viscous damping, in the case $AR > 0.15$ this coefficient depends mainly upon the area ratio of the pattern. The coefficient, N^* , entering into the expression for optimum number of holes, is sensitive also to the other geometrical parameters. Hence, it is possible that in the case of elongated holes the geometrical parameters other than AR may be used for optimizing the structure based on other criteria such as the number of holes, mechanical resistance, and penetration of sound waves. Particularly, in the case of microphones working in a natural environment, it is likely that a microplanar structure having oval holes will work better as a protecting surface against dust particles and water drops than a planar surface having circular holes.

The analysis of this paper is directed mainly to the acoustical domain. The results also can be used for designing purposes of MEMS in other applications such as accelerometers,^{3,5-7} micromechanical switches,⁸ various resonators,⁹ and tunable microoptical interferometers.¹⁰

II. SQUEEZE-FILM DAMPING

In order to study the viscous squeeze-film damping on a microphone, we model the air in the gap between the diaphragm and backplate (or between the diaphragm and the protecting structure) as an incompressible viscous gas. We refer the motion of the fluid at a Cartesian system whose origin is halfway between the plates average position and the xOy plane is parallel to the backplate surface.

A. Equations of the fluid dynamics

The complete system of equations for describing the motion of an incompressible viscous fluid consists of the continuity equation and the Navier-Stokes equations

$$\nabla \cdot \mathbf{v} = 0, \quad (1)$$

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) = \rho \mathbf{g} - \nabla p + \mu \nabla^2 \mathbf{v}, \quad (2)$$

where ρ is the density, μ is viscosity, p the pressure, \mathbf{v} denotes the velocity field, and \mathbf{g} is the gravity acceleration.

In the case of simple harmonic oscillations (of ω -angular frequency), we have

$$p = p_\omega e^{-i\omega t}, \quad \mathbf{v} = \mathbf{v}_\omega e^{-i\omega t}, \quad \frac{\partial p}{\partial t} = -i\omega p.$$

The system of equations (1) (2) becomes

$$\nabla \cdot \mathbf{v}_\omega = 0, \quad (3)$$

$$-i\omega \rho \mathbf{v}_\omega + \rho (\mathbf{v} \cdot \nabla) \mathbf{v}_\omega = \rho \mathbf{g} \exp(i\omega t) - \nabla p_\omega + \mu \nabla^2 \mathbf{v}_\omega. \quad (4)$$

We consider now dimensionless variables. As the domain in our case is the narrow air gap between the diaphragm and backplate, we will use different scales on the x , y , and z directions

$$x = L_0 x', \quad y = L_0 y', \quad z = d_0 z',$$

$$v_{\omega x} = V_0 v'_x, \quad v_{\omega y} = V_0 v'_y, \quad v_{\omega z} = \epsilon V_0 v'_z,$$

$$p_\omega - p_a e^{i\omega t} = P_0 p',$$

d_0 being the distance between the plates at equilibrium, L_0 a characteristic length of the plane domain, V_0 a reference velocity, and $\epsilon = d_0/L_0$ a small parameter. We drop the primes and try to remember that we are now working in dimensionless variables. To the lower order in ϵ we obtain the equations corresponding to the lubrication approximation¹²

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0, \quad (5)$$

$$\frac{\partial^2 v_x}{\partial z^2} + iK^2 v_x = \frac{\partial p}{\partial x}, \quad (6)$$

$$\frac{\partial^2 v_y}{\partial z^2} + iK^2 v_y = \frac{\partial p}{\partial y}, \quad (7)$$

$$\frac{\partial p}{\partial z} = 0. \quad (8)$$

We have denoted

$$P_0 = \frac{\mu V_0 L_0}{d_0^2}, \quad K = d_0 \sqrt{\frac{\rho_0 \omega}{\mu}}. \quad (9)$$

B. The boundary conditions for the velocity

Since we consider the gas as a viscous fluid, the appropriate boundary condition is the sticking of fluid particles to the solid walls. Hence, in dimensionless variables the boundary conditions on the solid walls (the membrane and the backplate) have the form

$$v_x(x, y, \pm \frac{1}{2}) = 0, \quad v_y(x, y, \pm \frac{1}{2}) = 0, \quad (10)$$

$$v_z(x, y, -\frac{1}{2}) = 0, \quad v_z(x, y, \frac{1}{2}) = w. \quad (11)$$

By w we denote the Oz component of the velocity of the diaphragm, assumed a known quantity.

C. The representation of the velocity field

Equation (8) shows that within the lubrication approximation the pressure is constant across the gap depending only on x and y . Therefore, Eqs. (6) and (7) can be integrated and the integration constants can be determined by using the boundary conditions (10). There results

$$v_x(x,y,z) = \left(\frac{\cos\left(\frac{(1+i)}{\sqrt{2}}Kz\right)}{\cos\left(\frac{(1+i)}{2\sqrt{2}}K\right)} - 1 \right) \frac{i}{K^2} \frac{\partial p}{\partial x}, \quad (12)$$

$$v_y(x,y,z) = \left(\frac{\cos\left(\frac{(1+i)}{\sqrt{2}}Kz\right)}{\cos\left(\frac{(1+i)}{2\sqrt{2}}K\right)} - 1 \right) \frac{i}{K^2} \frac{\partial p}{\partial y}. \quad (13)$$

Equation (5) becomes

$$\frac{\partial v_z}{\partial z} = \left(1 - \frac{\cos\left(\frac{(1+i)}{\sqrt{2}}Kz\right)}{\cos\left(\frac{(1+i)}{2\sqrt{2}}K\right)} \right) \frac{i}{K^2} \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right).$$

The component v_z results by integrating this equation and using the first condition (11)

$$v_z = \left(z + \frac{1}{2} - \frac{\sin\left(\frac{(1+i)}{\sqrt{2}}Kz\right)}{\left(\frac{(1+i)}{\sqrt{2}}K\right) \cos\left(\frac{(1+i)}{2\sqrt{2}}K\right)} - \frac{\tan\left(\frac{(1+i)}{2\sqrt{2}}K\right)}{\left(\frac{(1+i)}{\sqrt{2}}K\right)} \right) \frac{i}{K^2} \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right). \quad (14)$$

Thus, we succeed in obtaining a representation of the velocity field of the air in the gap by means of the pressure function, which is the main unknown function of the problem.

D. The pressure equation

From the second of Eqs. (11), on the upper plate, $z = 1/2$, the velocity component v_z equals the value w defined by the motion of the diaphragm. Thus, the relationship (14) yields the equation for the pressure field in the form

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 12Mw, \quad (15)$$

where we have denoted

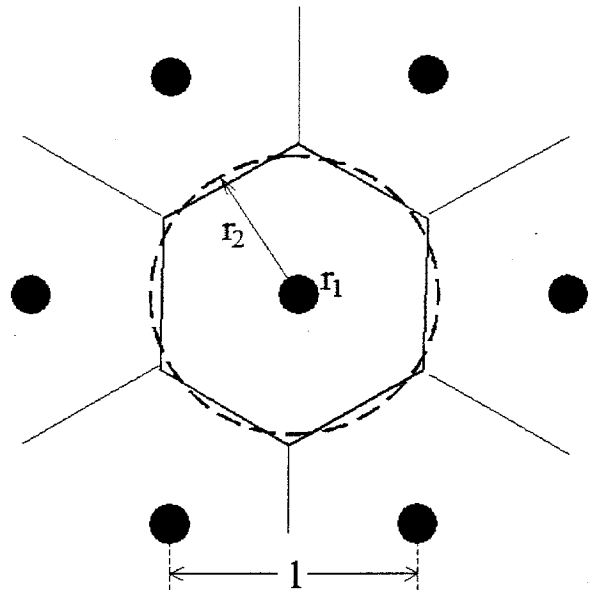


FIG. 1. The cell corresponding to a regular web of circular holes and its equivalent circular cell.

$$M = \frac{i}{12} K^2 \left(\frac{\tan\left(\frac{(1+i)}{2\sqrt{2}}K\right)}{\frac{(1+i)}{2\sqrt{2}}K} - 1 \right)^{-1}.$$

Equation (15) is a Poisson-type equation. The frequency enters only in the right-side term by means of the parameter M . This will give the same dependence upon frequency for the pressure and also for the forces resulting by pressure integration.

In the case of small values of K , a series expansion yields

$$M = 1 - \frac{id_0^2 \rho_0 \omega}{10\mu} + O\left(\frac{K^4}{100}\right).$$

A simple dimensional analysis reveals that for the case of air and frequencies between 100 Hz and 20 kHz the value $M = 1$ is a very good approximation.

E. Boundary conditions for the pressure

We suppose that on the backplate there is a periodic system of holes, as shown in Fig. 1, and define a cell C as the space occupied by a hole and its surrounding web space (the plane region where the hole is collecting the flow). By \mathcal{D}^h we denote the domain of the hole and by \mathcal{D} the domain of the cell corresponding to the plate region ($\mathcal{D} = C - \mathcal{D}^h$), and, finally, \mathcal{D}' is the domain resulting from \mathcal{D} by similarity transformation given by the scaling relationship.

On the holes we have the atmospheric pressure. This gives the condition

$$p = 0, \quad (16)$$

on the rim C_D of the holes. In the case we consider also the holes resistance—the pressure on the rim of the hole equals a constant (unknown) value p_1 .

In order to avoid considering a boundary-value problem for the plane domain external to the infinite number of holes, we will take advantage of the condition on the symmetry lines. Also, on the all symmetry lines for the pressure C_N in the xOy plane we have the condition

$$\frac{\partial p}{\partial n} = 0. \quad (17)$$

F. Example: Squeeze-film damping for the circular cells

We consider, for the beginning, the case where the domain D is an annulus of $r_1 < r_2$ radii. We take $L_0 = r_2$ and denote $r_0 = r_1/r_2$. Equation (15) becomes, in polar coordinates

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p}{\partial r} \right) = 12Mw, \quad \text{for } r_0 < r < 1, \quad (18)$$

and the associated boundary conditions can be written as

$$p(r_0) = 0, \quad \frac{\partial p}{\partial r}(1) = 0. \quad (19)$$

The solution of Eq. (18) satisfying the conditions (19) can be written as

$$p(r) = 12Mw \left(\frac{r^2}{4} - \frac{r_0^2}{4} - \ln \sqrt{\frac{r}{r_0}} \right).$$

We have also

$$\int \int_{\mathcal{D}} p \, dx dy = 12\pi M w \left(\frac{3}{8} - \frac{r_0^2}{2} + \frac{r_0^4}{8} + \frac{1}{2} \ln r_0 \right).$$

Finally, the force on the basic domain D can be written as

$$F^c = 12 \frac{\pi \mu r_2^4}{d_0^3} M \left(\frac{1}{2} \left(\frac{r_1}{r_2} \right)^2 - \frac{1}{8} \left(\frac{r_1}{r_2} \right)^4 - \frac{1}{2} \ln \frac{r_1}{r_2} - \frac{3}{8} \right) w^{\text{phys}}. \quad (20)$$

In the case of small values of K in Eq. (9) (this means in fact for not very high frequencies), the resulting expression coincides with the formula given by Škvor in Ref. 2

$$F^S = \frac{12\pi\mu r_2^4}{d_0^3} \left(\frac{1}{2} \left(\frac{r_1}{r_2} \right)^2 - \frac{1}{8} \left(\frac{r_1}{r_2} \right)^4 - \frac{1}{2} \ln \frac{r_1}{r_2} - \frac{3}{8} \right) w^{\text{phys}} \quad (21)$$

1. Remark

The obtained formulas for an annulus do not directly represent the array of holes typically used in designing a backplate. It is still possible to consider the external circle as an approximation of a regular hexagon (of the same area) and as a result, formula (21) can be used in the case of circular holes uniformly located in the corners of some squares or regular triangles for approximate calculation of the force F^S over an elementary cell.

III. THE HOLES' RESISTANCE

Some differences have been reported between the theoretically determined values of the viscous damping and the experimentally measured values.¹¹ One source of the errors is considered as being the zero-pressure condition on the rim of the holes. In the case where the thickness h of the backplate is comparable to the gap thickness d_0 , the resistance of the holes becomes an important component of the total viscous damping.

A. The general case

In order to determine the "holes' resistance" we assume a constant pressure p_1 along the upper edge of the hole and model the motion in the hole as a Poiseuille flow in a pipe driven by the pressure gradient

$$\frac{\partial p}{\partial z} = -\frac{p_1}{h}.$$

In this case the only nonvanishing component of velocity is v_z ,¹² and we can write the equation

$$\Delta v_z = \frac{p_1}{\mu h} \quad \text{in } \mathcal{D}^h, \quad (22)$$

with the *Dirichlet-type* boundary condition

$$v_z = 0, \quad \text{along } C_D. \quad (23)$$

The rate of flow in the domain \mathcal{D}^h can be written as

$$Q = \int \int_{\mathcal{D}^h} v_z(x, y) \, dx dy = \frac{p_1}{\mu h} A^h Q^0,$$

where A^h is the area of the domain \mathcal{D}^h , and Q^0 being a coefficient determined only by the geometry of the hole.

By equating this rate of flow with the air leaving the space between the diaphragm and backplate $Q^s = A w^{\text{phys}}$ (by A we have denoted the area of the domain \mathcal{D}), there results the value of the pressure p_1 as

$$p_1 = \frac{\mu h A}{A^h Q^0} w^{\text{phys}}.$$

Now, the supplementary force F^h for the cell C , due to hole resistance, can be written as

$$F^h = \frac{\mu h A^2}{A^h Q^0} w^{\text{phys}}. \quad (24)$$

B. Example: The case of circular holes

In the case of a circular hole of r_1 —radius, Eq. (22) becomes

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dv_z}{dr} \right) = \frac{p_1}{\mu h}, \quad \text{for } 0 \leq r < r_1.$$

The solution satisfying the condition (23) is

$$v_z(r) = \frac{p_1}{4\mu h} (r^2 - r_1^2),$$

and, correspondingly, the rate of flow can be written as

$$Q = \frac{\pi p_1}{8\mu h} r_1^4.$$

In this case there results $Q^0 = r_1^2/8$ and, correspondingly, the supplementary force for a cell is

$$F^h = 8\pi\mu h \frac{r_2^4}{r_1^4} w^{\text{phys}}. \quad (25)$$

C. The optimum number of circular holes

Let us denote by N the number of circular holes of a unit of area u^2 . Then, we have

$$\pi r_2^2 N = u^2, \quad r_1^2/r_2^2 = \text{AR}.$$

By considering the formulas (21) and (25) for the squeeze-film damping over a cell and the hole resistance, we can determine¹³ an optimum number of holes for a given value of AR, which assures an equilibrium between squeeze film and holes resistance

$$N_{\text{opt}} = \sqrt{\frac{3}{2hd_0^3} \frac{(\text{AR})}{\pi} \left[\frac{1}{2}(\text{AR}) - \frac{1}{8}(\text{AR})^2 - \frac{1}{4} \ln(\text{AR}) - \frac{3}{8} \right]^{1/2}} u^2,$$

and the associated minimum value of the total damping coefficient $B = F/w^{\text{phys}}$ as

$$B_{\text{min}} = \frac{8\sqrt{6}\mu}{(\text{AR})} \sqrt{\frac{h}{d_0^3} \left[\frac{1}{2}(\text{AR}) - \frac{1}{8}(\text{AR})^2 - \frac{1}{4} \ln(\text{AR}) - \frac{3}{8} \right]^{1/2}} u^2.$$

In the next sections we shall extend these results for two different geometries of the holes: the case of the slits between solid strips and the case of oval holes.

IV. THE CASE OF PLANAR MICROSTRUCTURES CONTAINING SLITS

In some cases for protection of microphones planar microstructures containing periodic slits can be used, i.e., the limit case of holes as long gaps between rectilinear strips. This type of structure can also be used as the second electrode in the case of condenser microphones. In fact, planar microstructures containing parallel slits have been used for fabricating micromachined capacitive accelerometers.³

We consider the geometry in Fig. 2 containing a periodic structure of $2L_0$ —width strips separated by $2r$ —wide slits.

A. The squeeze-film damping.

In the case where the domain \mathcal{D} is a strip parallel to Ox -axis, $0 < y < L_0$, we take L_0 as the reference length. Equation (15) becomes

$$\frac{d^2 p}{dy^2} = 12Mw, \quad 0 < y < L_0. \quad (26)$$

The boundary conditions become

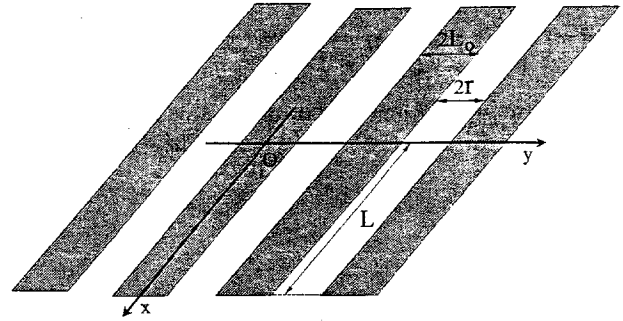


FIG. 2. A microstructure containing slits.

$$p(1) = 0, \quad (27)$$

$$\frac{dp}{dy}(0) = 0. \quad (28)$$

The solution of Eq. (26) satisfying the boundary conditions (27), (28) is

$$p(x) = 6M(y^2 - 1)w.$$

The force on a rectangle of L —length of the strip can be obtained by integration in the form

$$F^s = 8M\mu L \frac{L_0^3}{d_0^3} w^{\text{phys}}. \quad (29)$$

B. The slit's resistance

Equation (22) becomes in this case

$$\frac{d^2 v_z}{dy^2} = \frac{p_1}{\mu h}, \quad L_0 < y < L_0 + 2r, \quad (30)$$

with the boundary conditions on the slit sides

$$v_z(L_0) = 0, \quad v_z(L_0 + 2r) = 0. \quad (31)$$

Then, the solution of Eq. (30) satisfying the conditions (31) can be written as

$$v_z(x) = \frac{p_1}{2\mu h} (y - L_0)(y - L_0 - 2r).$$

The rate of flow corresponding to this velocity on a section of length L of the slit is

$$Q = \frac{2p_1 r^3}{3\mu h} L.$$

There results $Q^0 = r^2/3$ and formula (24) yields the hole (slit) resistance as

$$F^h = \frac{6\mu h (L_0 + r)^2}{r^3} L w^{\text{phys}}. \quad (32)$$

C. Optimal strip thickness and designing relationships

The total mechanical resistance on a backplate strip (of L width) due to viscosity can be obtained by adding the resistance due to squeezing film (29) with the slit resistance given by formula (32)

$$R = \left(\frac{8L_0^3}{d_0^3} + 6(L_0 + r)^2 \frac{h}{r^3} \right) \mu L. \quad (33)$$

We denote now by N the number of the slits on a unit length u and by AR the area ratio of the plate. Therefore

$$L_0 + r = \frac{u}{2N}, \quad (34)$$

$$r = u \frac{(\text{AR})}{2N}. \quad (35)$$

Formula (33) gives the damping coefficient \mathcal{B} on L —width of the backplate, as

$$\mathcal{B} \equiv NR = \mu L \left(\frac{(1 - (\text{AR}))^3 u^3}{d_0^3} + \frac{12h}{(\text{AR})^3} \frac{N^2}{u} \right).$$

This expression has a minimum value

$$\mathcal{B}_{\min} = 4\sqrt{3}\mu \left(\frac{1 - (\text{AR})}{(\text{AR})} \right)^{3/2} \sqrt{\frac{h}{d_0^3}} Lu,$$

corresponding to the value of N

$$N_{\text{opt}} = \frac{[(\text{AR})(1 - (\text{AR}))]^{3/4}}{\sqrt[4]{12hd_0^3}} u. \quad (36)$$

The relationship (36) determines the optimum number of parallel slits in terms of the area ratio AR, backplate thickness h , and average distance d_0 between backplate and diaphragm. Once N_{opt} is determined the relationships (34) and (35) provide the optimum strip and slit widths

$$L_{\text{opt}} = \frac{1 - (\text{AR})}{2N_{\text{opt}}} u$$

$$r_{\text{opt}} = \frac{(\text{AR})}{2N_{\text{opt}}} u.$$

1. Example

Let us consider as an example: $u = 1$ mm, $\text{AR} = 0.2$, $d_0 = 0.005$ mm, $h = 0.004$ mm. There results

$$N_{\text{opt}} = 29/\text{mm}, \quad L_{\text{opt}} = 0.014 \text{ mm},$$

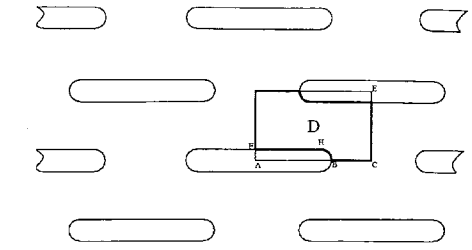
$$r_{\text{opt}} = 0.0035 \text{ mm}, \quad L = 1 \text{ mm},$$

$$\mathcal{B}_{\min}^0 = 0.1785 \times 10^{-3} \text{ N s/m}.$$

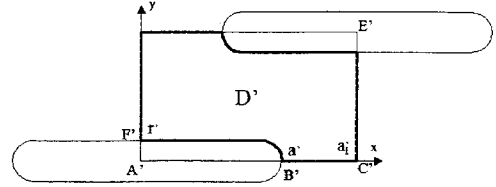
V. NUMERICAL SIMULATION OF VISCOUS DAMPING: THE CASE OF OVAL HOLES

A. The squeeze-film damping in the case of periodic oval holes

We consider the backplate structure in Fig. 3(a). It consists of a periodic pattern of offset oval holes. (By oval we mean the geometrical form of a rectangle with two half-circles of r radius added to smaller sides.) The basic domain, D , we are using to determine the pressure is also shown in Fig. 3(a) and the corresponding canonical domain D' resulting from D by similarity transformation given by the scaling relationship, is also drawn in Fig. 3(b).



(a)



(b)

FIG. 3. (a) An uniform offset system of oval holes. (b) The basic domain corresponding to the planar microstructure in (a).

We take as the scaling length the distance L_0 between two neighbor rows of holes. Consequently, we have in the domain D' , $C'E' = 1$, and denote: $A'C' = a'_1$, $A'B' = a'$, $A'F' = r'$, $r'L_0 = r$. We denote again by AR the area ratio of the plate (the fraction of the hole area to the cell area), and hence

$$4r(a - r) + \pi r^2 = (\text{AR})A.$$

The physical pressure in domain D can be obtained by considering the physical variables in Sec. IIA and equation in Sec. IID

$$p^{\text{phys}}(x, y) = -12 \frac{\mu L_0^2}{d_0^3} \hat{p} \left(\frac{x}{L_0}, \frac{y}{L_0} \right) w^{\text{phys}}.$$

$\hat{p}(x, y)$ denotes the solution of the boundary-value problem

$$\Delta \hat{p}(x, y) = 1, \quad \text{in } D', \quad (37)$$

$$\hat{p}(x, y) = 0 \quad \text{on } C'_D,$$

$$\frac{\partial \hat{p}(x, y)}{\partial n} = 0 \quad \text{on } C'_N. \quad (38)$$

Here, C'_D is the part of the boundary of the domain D' where the pressure is known (the rim of the holes) and C'_N the part of the boundary-containing segment of symmetry lines.

The viscous force on a cell given by the squeezing-film damping can be expressed as

$$F_{\text{cell}}^S = 24\mu \frac{L_0^4}{d_0^3} M a'_1 C_p w^{\text{phys}}, \quad (39)$$

where the pressure coefficient C_p has the expression

$$C_p = - \int \int_{D'} \hat{p}(x', y') dx' dy' / \int \int_{D'} dx' dy'. \quad (40)$$

Thus, the determination of the squeezing-film damping on a cell requires the solving of the boundary-value problem (37) and (38) and calculation of the pressure coefficient C_p by using formula (40).

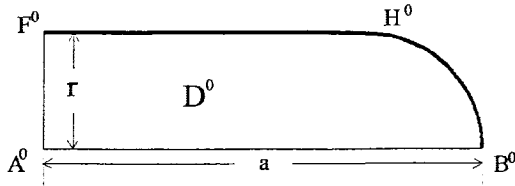


FIG. 4. The domain D^0 considered in determination of the resistance of the oval holes.

B. The resistance of oval holes

In order to determine the oval holes' resistance we consider the domain D^0 in Fig. 4 corresponding to a quarter fraction of hole, scaled by the width r . In this case the rate of flow through the hole can be obtained as

$$Q = \int \int_{D^h} v_z(x, y) dx dy = \frac{p_1}{\mu h} r^2 \mathcal{A}^h Q^0 \left(\frac{a}{r} \right), \quad (41)$$

where \mathcal{A}^h is the area of an oval hole and Q^0 is

$$Q^0 \left(\frac{a}{r} \right) = - \int \int_{D^{h0}} \hat{v}_z(x', y') dx' dy' / \int \int_{D^{h0}} dx' dy'. \quad (42)$$

The function $\hat{v}_z(x, y)$ is the solution of the Poisson equation

$$\Delta \hat{v}_z(x, y) = 1, \quad \text{in } D^{h0}, \quad (43)$$

satisfying the boundary conditions

$$\begin{aligned} \hat{v}_z(x, y) &= 0, \quad \text{along } F^0 H^0 B^0, \\ \frac{\partial \hat{v}_z}{\partial x} &= 0, \quad \text{along } F^0 A^0, \\ \frac{\partial \hat{v}_z}{\partial y} &= 0, \quad \text{along } A^0 B^0. \end{aligned} \quad (44)$$

Now, the supplementary force due to the oval hole resistance F^h can be written as

$$F^h = \frac{\mu h \mathcal{A}^2}{r^2 Q^0(a/r) \mathcal{A}^h} w^{\text{phys}}. \quad (45)$$

The function Q^0 has been determined numerically. We preferred a finite element approach in an effort to use programs familiar to the solid mechanics researchers. In order to model the Poisson's equation we used ANSYS 2D steady heat-transfer elements. Thus, by solving numerically the boundary-value problem (43), (44), and computing the integral in (42), there results the function plotted in Fig. 5. The two limit cases have definite physical meanings: for $a=r$ we have the case of circular holes in which case $Q^0=1/8$, while for $a \rightarrow \infty$ we obtain the slit case where $Q^0=1/3$ as in Secs. III B and IV B.

C. The optimal number of oval holes for a planar microstructure

Adding the squeeze-film damping with the oval hole resistance results in the total viscous force on a cell in the form

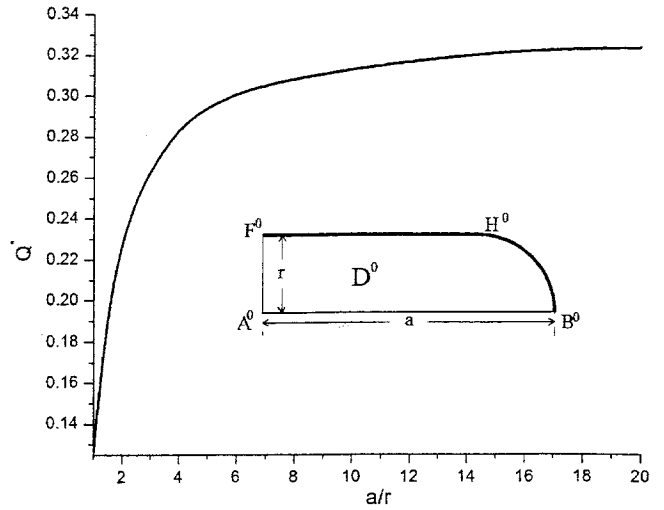


FIG. 5. The function Q^0 for determining the viscous resistance of the oval holes.

$$F_{\text{cell}}^T = \left(\frac{24 \mu L_0^4 M a_1'}{d_0^3} C_p + \frac{\mu h \mathcal{A}^2}{r^2 \mathcal{A}^h} \frac{1}{Q^0(a/r)} \right) w^{\text{phys}}.$$

Let us now denote by N the number of holes of a unit of area u^2 . Then we have

$$\mathcal{A} = \frac{u^2}{N}, \quad \mathcal{A}^h = (\text{AR}) \cdot \mathcal{A}, \quad \mathcal{A} = 2a_1' L_0^2$$

$$\frac{1}{r^2} = \frac{2a_1'^2}{a_1' (\text{AR})^2 u^2} R^2 N, \quad k \equiv \frac{a'}{a_1'}$$

$$R \equiv 1 + \sqrt{1 - \frac{4 - \pi}{2} \frac{a_1'}{a'^2} (\text{AR})}.$$

The total force on u^2 area results in the form

$$F^T = \left(\frac{6 M C_p}{a_1' d_0^3} \frac{u^4}{N} + \frac{2 h a_1'^2}{a_1'} \frac{R^2}{(\text{AR})^3 Q^0(a/r)} N \right) \mu w^{\text{phys}}.$$

The total viscous damping force has a minimum value for $N = N_{\text{opt}}$

$$N_{\text{opt}} = \frac{10^{-4} \sqrt{M} u^2}{\sqrt{h d_0^3}} N^*. \quad (46)$$

The corresponding damping coefficient $B_{\text{min}} = F_{\text{min}}^T / w^{\text{phys}}$ can be written as

$$B_{\text{min}} = \sqrt{\frac{48 h M}{d_0^3}} \mu u^2 B^*. \quad (47)$$

The numerical coefficients N^* and B^* depend only on the geometry of the oval holes

$$N^* = \frac{\sqrt{3 C_p (\text{AR})^3 Q^0(a/r)}}{a' R} 10^4, \quad (48)$$

$$B^* = \frac{a'}{a_1'} \sqrt{\frac{C_p}{(\text{AR})^3 Q^0(a/r)}} R. \quad (49)$$

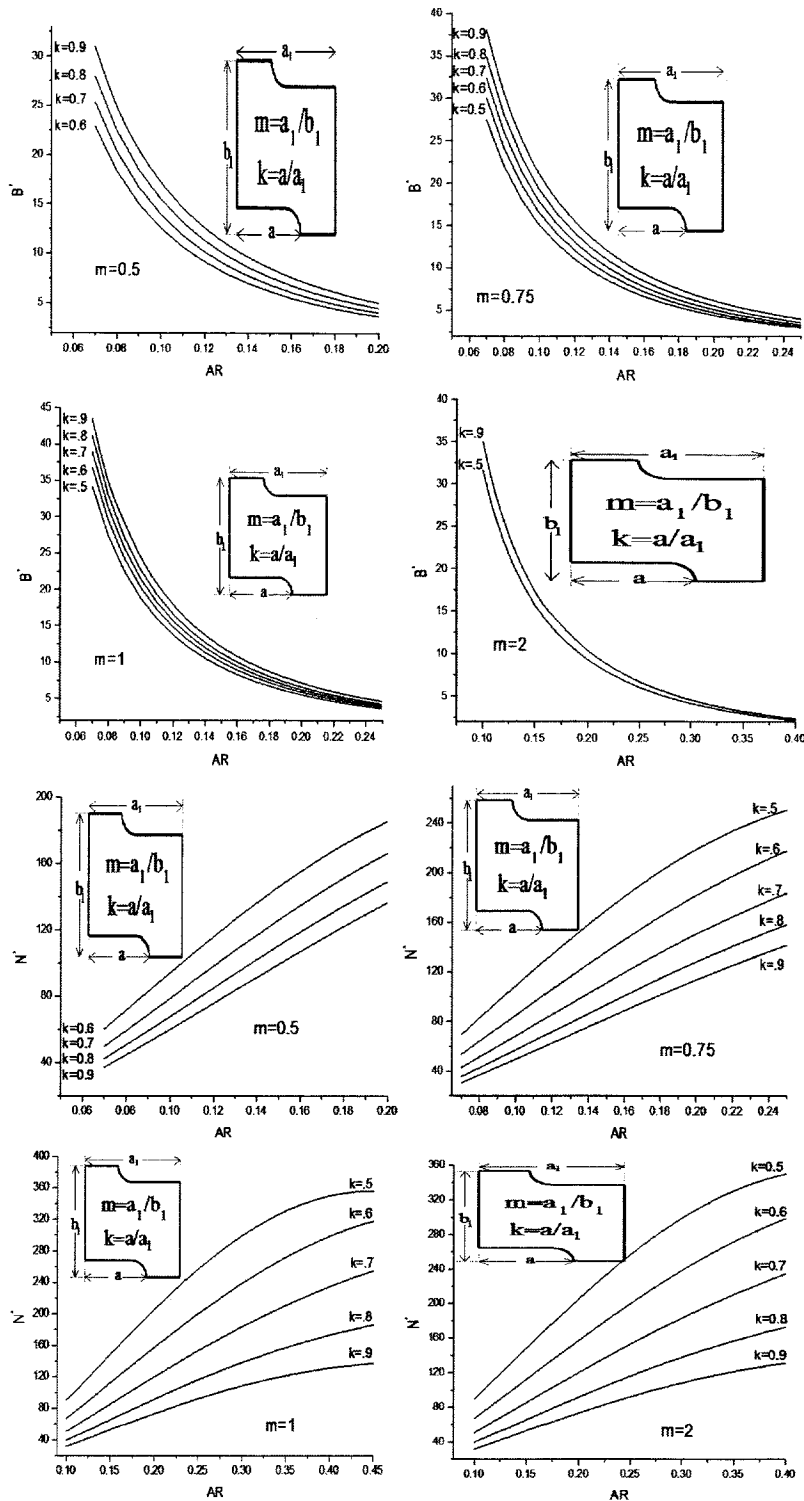


FIG. 6. (a) The numerical coefficient B^* for calculating the minimum damping coefficient for several dimensions of the basic domain (m) and holes (k). (b) The numerical coefficient N^* for determining the optimum number of holes for several dimensions of the basic domain (m) and holes (k).

D. Numerical results and designing relationships

The formulas (48) and (49) for determining the numerical coefficients N^* and B^* have been applied for certain hole geometries described by the dimensions of the domain D^* : $m \equiv a_1'$, $k \equiv a'/a_1'$, and for a sequence of values of area ratio AR between 0.08 and 0.4.

The boundary-value problem (37) and (38) has to be solved many times; this is why we preferred a more efficient complex variable boundary element method¹⁴ which is doing all the computation only on the boundary curve.

The graphs of the coefficients B^* and N^* are presented in Figs. 6(a) and (b), respectively. It is evident that for small values of the area ratio all the geometrical parameters of the oval pattern are important in determining the minimum value of viscous damping coefficient. In the case $AR > 0.15$ the variation of the coefficient B^* with k is very slow. Therefore, the value of this coefficient can be adjusted by choosing the proper value of hole area ratio AR . On the other hand, the coefficient N^* is very sensitive to values of k ; thus, it is possible by choosing this parameter properly to obtain a per-

forated planar microstructure satisfying other designing demands.

The presented graphs involve only the geometrical parameters of the oval holes: AR, $m = a'_1$, $k = a'/a'_1$. Consequently, they can be used for calculating the damping coefficient and the optimum number of holes for different distances between plates, backplate thicknesses, frequencies, and viscosities.

Once the values of the parameters AR, a'_1 , and a' are decided (and implicitly the values of numerical coefficients B^* and N^*), the number of holes is given by the relationship (46) and the physical dimensions of the cell by formulas

$$b_{1o} = \frac{u}{\sqrt{2a'_1 N_{opt}}},$$

$$a_{1o} = a'_1 b_{1o}, \quad a_o = a' b_{1o},$$

$$r_o = \frac{2a_o}{4 - \pi} \left(1 - \sqrt{1 - \frac{(2 - \pi/2)(AR)u}{a_o N_{opt}}} \right).$$

VI. CONCLUSIONS

In the case of small area ratio the optimal geometry of the holes of planar microstructures is determined by all the geometrical parameters of the structure.

For $AR > 0.15$ the influence of the other geometrical parameters other than AR on the damping coefficient is very slight. Therefore, it is possible to adjust the holes pattern to fulfill other designing criteria as number of holes, structure resistance, penetration of sound waves, etc. For example, a protecting surface having oval holes rather than circular ones will be more appropriate for protecting a microphone diaphragm against dust particles and water drops.

The designing formulas in the case of oval holes contain the constants B^* and N^* depending only on the geometry of the holes. These coefficients can be computed by solving the Poisson's equation (with certain mixed boundary conditions) by using the appropriate software. Otherwise, the coefficients can be evaluated by using the graphs provided in the paper. Once B^* and N^* are determined, the total viscous damping and the optimum number of holes can be obtained for vari-

ous backplate thicknesses, distances between plates, frequencies, and viscosities.

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